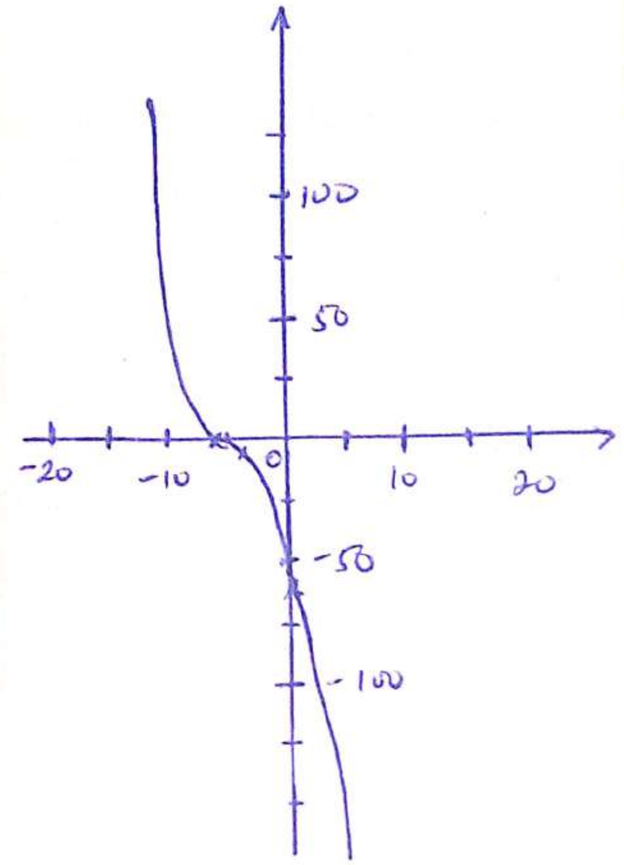
Math. 1314 Test 1.

1. a) $f(x) = -(x+4)^3 + 2$



b). Reflection in the x-axis; horizontal shift 4 units left; Vertical shift 2 unit up.

c) Domain.

$-\infty < x < \infty$

Interval notation: $(-\infty, \infty)$

d) Range.

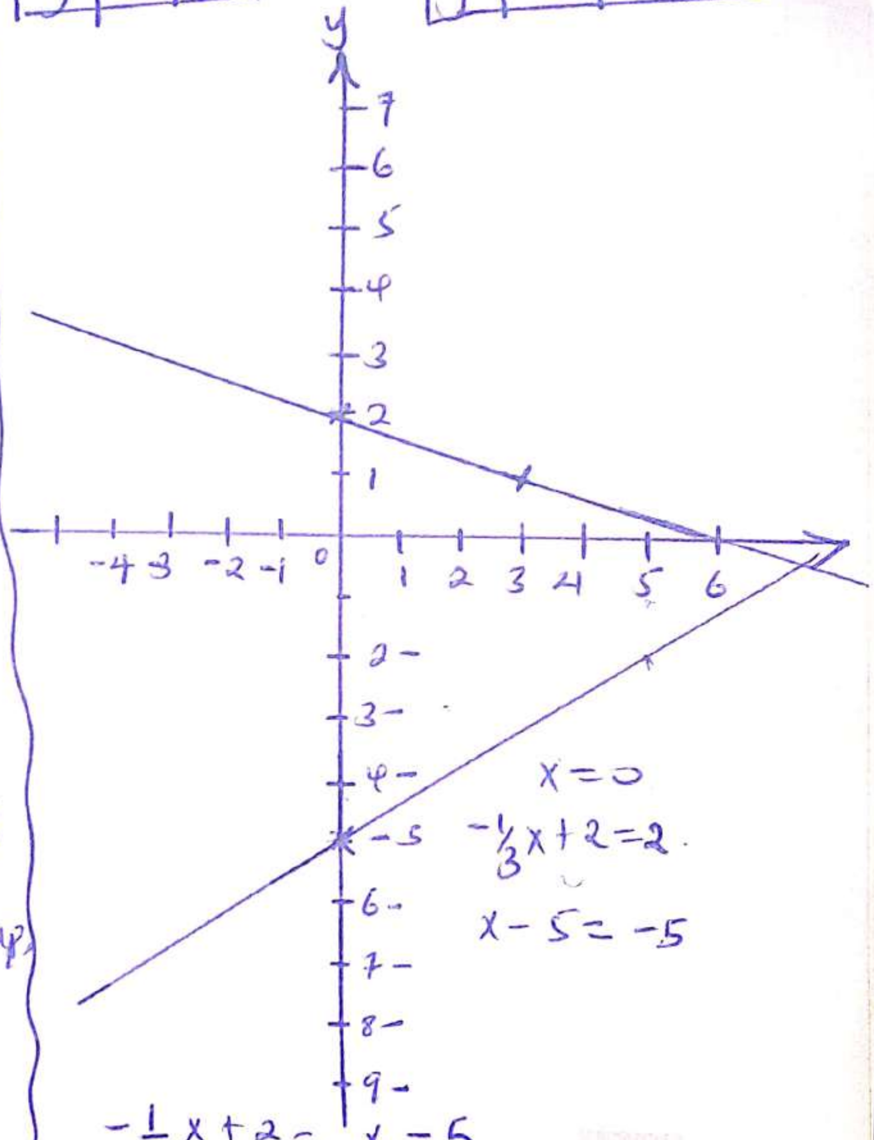
$-\infty < f(x) < \infty$

Interval notation: $(-\infty, \infty)$

2. a) $f(x) = \begin{cases} -\frac{1}{3}x + 2, & x \le 0 \\ x - 5, & x > 0 \end{cases}$

x	0	3
y	2	1

x	0	2	3
y	-5	-3	-2



$x = 0$

$-\frac{1}{3}x + 2 = 2$

$x - 5 = -5$

$-\frac{1}{3}x + 2 = x - 5$

$-x + 6 = 3x - 15$

$-4x = -21$

$x = \frac{21}{4}$.

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$.

$f(x) = x^2 - 5x + 2$

$\dfrac{f(x+h) - f(x)}{h}$

$f(x+h) = (x+h)^2 - 5(x+h) + 2$

$= x^2 + 2xh + h^2 - 5x - 5h + 2.$

$f(x) = x^2 - 5x + 2$

$f(x+h) - f(x) = x^2 + 2xh + h^2 - 5x - 5h + 2 - (x^2 - 5x + 2)$

$= \cancel{x^2} + 2xh + h^2 - \cancel{5x} - 5h + \cancel{2} - \cancel{x^2} + \cancel{5x} - \cancel{2}$

$= 2xh + h^2 - 5h$

$= h(2x + h - 5)$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{\cancel{h}(2x+h-5)}{\cancel{h}}$

difference quotient is $(2x + h - 5)$

4. $f(x) = 7(x^2 + 10).$

$f(x) = x^2.$

y-axis

$f(x) = x^2.$

horizontal shift 10 units left.

$f(x) = (x+10)^2$

vertical shift 12 units up

$f(x) = (x+10)^2 + 12.$

vertical stretch by 7.

$f(x) = 7(x+10)^2 + 12.$

$f(x) = 7(x+10)^2 + 12.$

5) $f(x) = \dfrac{6}{x}$, $g(x) = \dfrac{1}{2x+1}$

a) $(f \circ g)(x) = (f(g(x))$

$= \dfrac{6}{\frac{1}{2x+1}} = 6(2x+1)$

$(f \circ g)(x) = 12x + 6.$

b) $(g \circ f)(x) = (g(f(x))$

$= \dfrac{1}{2(6/x)+1} = \dfrac{1}{\frac{12}{x}+1}$

$\dfrac{1}{\frac{12+x}{x}}$

$(g \circ f)(x) = \dfrac{x}{12+x}.$

c) Domain.

$12x + 6 = 0$

$12x = -6$

$x = -\frac{1}{2}.$

$\therefore x < -\frac{1}{2}$

$x > -\frac{1}{2}$

$(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$ (B).

d) Domain.

$12 + x = 0$

$x = -12.$

$x < -12, \; x > -12.$

$(-\infty, -12) \cup (-12, \infty)$ (A)

BONUS

a) Domain $(-\infty, \infty)$

b) Range $(-\infty, \infty)$.

c) Maxima $= 2, -3$

d) Minima $= -1, 4$.

e) local maxima $= 2$

g) $(-\infty, -3), (-1, 2) (4, \infty)$

h) $(-3, -1), (2, 4)$.